

Dérivées : solutions

Dérive les expressions suivantes :

a) $(4x)' = 4$

b) $(2x^2 - 5x + 7)' = 4x - 5$

c) $\left(-\frac{x}{3}\right)' = \frac{-1}{3}$

d) $(\sqrt{x} + 4x - 1)' = \left(x^{\frac{1}{2}} + 4x - 1\right)' = \frac{1}{2\sqrt{x}} + 4$

e)
$$\begin{aligned} \left(\frac{1-2x}{3x-1}\right)' &= \frac{(1-2x)' \cdot (3x-1) - (1-2x) \cdot (3x-1)'}{(3x-1)^2} = \frac{-2 \cdot (3x-1) - (1-2x) \cdot 3}{(3x-1)^2} \\ &= \frac{-6x + 2 - 3 + 6x}{(3x-1)^2} = \frac{-1}{(3x-1)^2} \end{aligned}$$

f) $(x + \cos x)' = 1 - \sin x$

g) $(3 \sin x - 2 \cos x)' = 3 \cos x + 2 \sin x$

h)
$$\begin{aligned} ((2+x)(1-5x))' &= (2+x)'(1-5x) + (2+x)(1-5x)' = 1(1-5x) + (2+x) \cdot (-5) = \\ &= 1 - 5x - 10 - 5x = -10x - 9 \end{aligned}$$

i)
$$\begin{aligned} ((x^4 - 1)(2x + 3))' &= (x^4 - 1)'(2x + 3) + (x^4 - 1)(2x + 3)' = 4x^3(2x + 3) + (x^4 - \\ &= 1) \cdot 2 = 8x^4 + 12x^3 + 2x^4 - 2 = 10x^4 + 12x^3 - 2 \end{aligned}$$

j)
$$\begin{aligned} ((x - x^2)(3x))' &= (x - x^2)' \cdot 3x + (x - x^2) \cdot (3x)' = (1 - 2x) \cdot 3x + (x - x^2) \cdot 3 = 3x - 6x^2 + \\ &= 3x - 3x^2 = -9x^2 + 6x \end{aligned}$$

k) $(x^2 \cdot \cos x)' = (x^2)' \cdot \cos x + x^2 \cdot (\cos x)' = 2x \cdot \cos x - x^2 \cdot \sin x$

l) $(\sin x \cdot \cos x)' = (\sin x)' \cdot \cos x + \sin x \cdot (\cos x)' = \cos x \cdot \cos x + \sin x \cdot (-\sin x) = \cos^2 x - \sin^2 x$

m)
$$\left(\frac{2x^3}{x^2-1}\right)' = \frac{(2x^3)' \cdot (x^2-1) - 2x^3 \cdot (x^2-1)'}{(x^2-1)^2} = \frac{6x^2 \cdot (x^2-1) - 2x^3 \cdot 2x}{(x^2-1)^2} = \frac{6x^4 - 6x^2 - 4x^4}{(x^2-1)^2} = \frac{2x^4 - 6x^2}{(x^2-1)^2}$$

n)
$$\begin{aligned} \left(\frac{4x^2+3x-1}{2x+1}\right)' &= \frac{(4x^2+3x-1)'(2x+1) - (4x^2+3x-1) \cdot (2x+1)'}{(2x+1)^2} = \frac{(8x+3)(2x+1) - (4x^2+3x-1) \cdot 2}{(2x+1)^2} = \\ &= \frac{16x^2+8x+6x+3-8x^2-6x+2}{(2x+1)^2} = \frac{8x^2+8x+5}{(2x+1)^2} \end{aligned}$$

o)
$$\begin{aligned} ((x^3 - 5x)(2x^2 - 4))' &= (x^3 - 5x)'(2x^2 - 4) + (x^3 - 5x)(2x^2 - 4)' = (3x^2 - \\ &= 5) \cdot (2x^2 - 4) + (x^3 - 5x) \cdot 4x = 6x^4 - 12x^2 - 10x^2 + 20 + 4x^4 - 20x^2 = 10x^4 - \\ &= 42x^2 + 20 \end{aligned}$$

p)
$$\begin{aligned} \left(x + \frac{x^2-1}{x^2+1}\right)' &= 1 + \frac{(x^2-1)' \cdot (x^2+1) - (x^2-1) \cdot (x^2+1)'}{(x^2+1)^2} = 1 + \frac{2x \cdot (x^2+1) - (x^2-1) \cdot 2x}{(x^2+1)^2} = 1 + \frac{2x^3+2x-2x^3+2x}{(x^2+1)^2} = \\ &= 1 + \frac{4x}{(x^2+1)^2} \end{aligned}$$

Dérivées de fonctions composées

$$\text{a) } ((3x + 4)^2)' = 2 \cdot (3x + 4) \cdot (3x + 4)' = 2 \cdot (3x + 4) \cdot 3 = 6 \cdot (3x + 4)$$

$$\text{b) } ((x - x^2)^3)' = 3 \cdot (x - x^2)^2 \cdot (x - x^2)' = 3 \cdot (x - x^2)^2 \cdot (1 - 2x)$$

$$\text{c) } ((2 - 4x)^3)' = 3 \cdot (2 - 4x)^2 \cdot (2 - 4x)' = 3 \cdot (2 - 4x)^2 \cdot (-4) = -12 \cdot (2 - 4x)^2$$

$$\begin{aligned} \text{d) } \left(\frac{3}{(2x+1)^2} \right)' &= (3 \cdot (2x+1)^{-2})' = 3 \cdot (-2) \cdot (2x+1)^{-3} \cdot (2x+1)' = -6 \cdot (2x+1)^{-3} \cdot 2 = \\ &= -12 \cdot (2x+1)^{-3} = \frac{-12}{(2x+1)^3} \end{aligned}$$

$$\begin{aligned} \text{e) } (\cos^3 2x)' &= ((\cos 2x)^3)' = 3 \cdot (\cos 2x)^2 \cdot (\cos 2x)' = 3 \cdot (\cos 2x)^2 \cdot (-2) \sin 2x = \\ &= -6 \cdot \cos^2 2x \cdot \sin 2x \end{aligned}$$

$$\text{f) } (\cos(2x - 4))' = -(2x - 4)' \cdot \sin(2x - 4) = -2 \cdot \sin(2x - 4)$$

$$\begin{aligned} \text{g) } \left(\left(\frac{1+x}{1-x} \right)^2 \right)' &= 2 \cdot \left(\frac{1+x}{1-x} \right) \cdot \left(\frac{1+x}{1-x} \right)' = 2 \cdot \left(\frac{1+x}{1-x} \right) \cdot \frac{(1+x)' \cdot (1-x) - (1+x) \cdot (1-x)'}{(1-x)^2} = \\ &= 2 \cdot \left(\frac{1+x}{1-x} \right) \cdot \frac{1 \cdot (1-x) - (1+x) \cdot (-1)}{(1-x)^2} = 2 \cdot \left(\frac{1+x}{1-x} \right) \cdot \frac{2}{(1-x)^2} = \frac{4 \cdot (1+x)}{(1-x)^3} \end{aligned}$$

$$\text{h) } \left(\left(1 + \frac{1}{x} \right)^3 \right)' = 3 \cdot \left(1 + \frac{1}{x} \right)^2 \cdot \left(1 + \frac{1}{x} \right)' = 3 \cdot \left(1 + \frac{1}{x} \right)^2 \cdot \left(\frac{-1}{x^2} \right) = \frac{-3}{x^2} \cdot \left(1 + \frac{1}{x} \right)^2$$

$$\begin{aligned} \text{i) } (\sin^2(3x + 2))' &= ((\sin(3x + 2))^2)' = 2 \cdot (\sin(3x + 2)) \cdot (\sin(3x + 2))' = \\ &= 2 \cdot \sin(3x + 2) \cdot 3 \cos(3x + 2) = 6 \cdot \sin(3x + 2) \cdot \cos(3x + 2) \end{aligned}$$

$$j) (\sqrt{3x})' = \left((3x)^{\frac{1}{2}} \right)' = \frac{1}{2} \cdot (3x)^{\frac{1}{2}-1} \cdot (3x)' = \frac{1}{2} \cdot (3x)^{-\frac{1}{2}} \cdot 3 = \frac{3}{2\sqrt{3x}}$$

$$k) (\operatorname{tg} 4x)' = \frac{(4x)'}{\cos^2 4x} = \frac{4}{\cos^2 4x}$$

$$l) (\sqrt{x^2 - 4x})' = ((x^2 - 4x)^{\frac{1}{2}})' = \frac{1}{2} \cdot (x^2 - 4x)^{\frac{1}{2}-1} \cdot (x^2 - 4x)' = \frac{1}{2} (x^2 - 4x)^{-\frac{1}{2}} \cdot (2x - 4) =$$

$$\frac{1}{2} \cdot (x^2 - 4x)^{-\frac{1}{2}} \cdot 2(x - 2) = \frac{x-2}{\sqrt{x^2-4x}}$$

$$m) ((3x^2 - 1)^3)' = 3 \cdot (3x^2 - 1)^2 \cdot (3x^2 - 1)' = 3 \cdot (3x^2 - 1)^2 \cdot 6x = 18x \cdot (3x^2 - 1)^2$$

$$n) \left(\frac{1}{\sqrt{3x^4 - 2x}} \right)' = \left((3x^4 - 2x)^{-\frac{1}{2}} \right)' = \frac{-1}{2} \cdot (3x^4 - 2x)^{-\frac{1}{2}-1} \cdot (3x^4 - 2x)'$$

$$= \frac{-1}{2} \cdot (3x^4 - 2x)^{-\frac{3}{2}} \cdot (12x^3 - 2) = \frac{-1}{2} \cdot (3x^4 - 2x)^{-\frac{3}{2}} \cdot 2 \cdot (6x^3 - 1)$$

$$= \frac{6x^3 - 1}{\sqrt{(3x^4 - 2x)^3}}$$

$$o) (\sin^6(3x - 5))' = (((\sin(3x - 5))^6)' = 6 \cdot (\sin(3x - 5))^5 \cdot (\sin(3x - 5))' =$$

$$6 \cdot (\sin(3x - 5))^5 \cdot 3 \cdot \cos(3x - 5) = 18 \cdot \sin^5(3x - 5) \cdot \cos(3x - 5)$$

Exercices variés :

$$a) (-6x^5 - 3x^4 + 2x - 1)' = -30x^4 - 12x^3 + 2$$

$$b) ((-5x^2 - 3x + 1)^3)' = 3 \cdot (-5x^2 - 3x + 1)^2 \cdot (-5x^2 - 3x + 1)' = \\ 3 \cdot (-5x^2 - 3x + 1)^2 \cdot (-10x - 3)$$

$$c) \left(\frac{3x^2 - 4x}{2 - 3x} \right)' = \frac{(3x^2 - 4x)' \cdot (2 - 3x) - (3x^2 - 4x) \cdot (2 - 3x)'}{(2 - 3x)^2} = \\ \frac{(6x - 4) \cdot (2 - 3x) - (3x^2 - 4x) \cdot (-3)}{(2 - 3x)^2} = \frac{12x - 18x^2 - 8 + 12x + 9x^2 - 12x}{(2 - 3x)^2} = \frac{-9x^2 + 12x - 8}{(2 - 3x)^2}$$

$$d) (\cos(2x - 4))' = -(2x - 4)' \cdot \sin(2x - 4) = -2 \cdot \sin(2x - 4)$$

$$e) (x^2 \cdot \sin 3x)' = (x^2)' \cdot \sin 3x + x^2 \cdot (\sin 3x)' = 2x \cdot \sin 3x + x^2 \cdot (3x)' \cdot \cos 3x = \\ 2x \cdot \sin 3x + 3x^2 \cdot \cos 3x$$

$$f) (5x^2 \cdot (2x + 1))' = (5x^2)' \cdot (2x + 1) + 5x^2 \cdot (2x + 1)' = 10x \cdot (2x + 1) + 5x^2 \cdot 2 = \\ 20x^2 + 10x + 10x^2 = 30x^2 + 10x$$

$$g) (x \cdot \sqrt{x})' = (x^1 \cdot x^{\frac{1}{2}})' = (x^{1+\frac{1}{2}})' = (x^{\frac{3}{2}})' = \frac{3}{2} \cdot x^{\frac{1}{2}} = \frac{3}{2} \sqrt{x}$$

$$h) \left(\frac{x^3 + 1}{x^3 - 1} \right)' = \frac{(x^3 + 1)' \cdot (x^3 - 1) - (x^3 + 1) \cdot (x^3 - 1)'}{(x^3 - 1)^2} = \frac{(3x^2) \cdot (x^3 - 1) - (x^3 + 1) \cdot 3x^2}{(x^3 - 1)^2} = \\ \frac{3x^5 - 3x^2 - 3x^5 - 3x^2}{(x^3 - 1)^2} = \frac{-6x^2}{(x^3 - 1)^2}$$

$$i) (\sqrt{2 - 3x})' = ((2 - 3x)^{\frac{1}{2}})' = \frac{1}{2} \cdot (2 - 3x)^{-\frac{1}{2}} \cdot (2 - 3x)' = \frac{1}{2} \cdot \frac{1}{(2 - 3x)^{\frac{1}{2}}} \cdot (-3) = \\ \frac{-3}{2 \cdot \sqrt{2 - 3x}}$$

$$j) (\cos x \cdot (\sin^2 x + 2))' = (\cos x)' \cdot (\sin^2 x + 2) + \cos x \cdot (\sin^2 x + 2)' = \\ -\sin x \cdot (\sin^2 x + 2) + \cos x \cdot 2 \sin x \cdot \cos x = -\sin x \cdot (\sin^2 x + 2) + 2 \sin x \cos^2 x$$

$$k) (\operatorname{tg}^2 3x)' = ((\operatorname{tg} 3x)^2)' = 2 \cdot \operatorname{tg} 3x \cdot (\operatorname{tg} 3x)' = 2 \cdot \operatorname{tg} 3x \cdot \frac{3}{\cos^2 3x} = 6 \frac{\operatorname{tg} 3x}{\cos^2 3x}$$

$$\begin{aligned}
 \text{l) } \left(\frac{x^2+x+1}{1-x} \right)' &= \frac{(x^2+x+1)' \cdot (1-x) - (x^2+x+1) \cdot (1-x)'}{(1-x)^2} = \frac{(2x+1) \cdot (1-x) - (x^2+x+1) \cdot (-1)}{(1-x)^2} = \\
 &= \frac{2x-2x^2+1-3+x^2+x+1}{(1-x)^2} = \frac{-x^2+3x+2}{(1-x)^2}
 \end{aligned}$$

$$\begin{aligned}
 \text{m) } ((4x^2 - 5x + 1) \cdot (2x - 3)^3)' &= (4x^2 - 5x + 1)' \cdot (2x - 3)^3 + (4x^2 - 5x + 1) \cdot \\
 &= 1 \cdot ((2x - 3)^3)' = (8x - 5) \cdot (2x - 3)^3 + (4x^2 - 5x + 1) \cdot 3 \cdot (2x - 3)^2 \cdot 2 = \\
 &= (8x - 5) \cdot (2x - 3)^3 + 6 \cdot (4x^2 - 5x + 1) \cdot (2x - 3)^2
 \end{aligned}$$

$$\begin{aligned}
 \text{n) } \left(\frac{2x-1}{(3x+2)^2} \right)' &= \frac{(2x-1)' \cdot (3x+2)^2 - (2x-1) \cdot ((3x+2)^2)'}{(3x+2)^4} = \frac{2 \cdot (3x+2)^2 - (2x-1) \cdot 2 \cdot (3x+2) \cdot 3}{(3x+2)^4} = \\
 &= \frac{2 \cdot (3x+2)^2 - 6 \cdot (2x-1) \cdot (3x+2)}{(3x+2)^4} = \frac{2 \cdot (3x+2) \cdot ((3x+2) - 3 \cdot (2x-1))}{(3x+2)^4} = \frac{2 \cdot (3x+2-6x+3)}{(3x+2)^4} = \\
 &= \frac{2 \cdot (-3x+5)}{(3x+2)^4}
 \end{aligned}$$

$$\begin{aligned}
 \text{o) } \left(\frac{(3x^2+2x)^2}{2x-3} \right)' &= \frac{((3x^2+2x)^2)' \cdot (2x-3) - (3x^2+2x)^2 \cdot (2x-3)'}{(2x-3)^2} = \\
 &= \frac{2 \cdot (3x^2+2x) \cdot (3x^2+2x)' \cdot (2x-3) - (3x^2+2x)^2 \cdot 2}{(2x-3)^2} = \\
 &= \frac{2 \cdot (3x^2+2x) \cdot (6x+2) \cdot (2x-3) - 2 \cdot (3x^2+2x)^2}{(2x-3)^2} = \frac{2 \cdot (3x^2+2x) \cdot ((6x+2) \cdot (2x-3) - 1)}{(2x-3)^2} = \\
 &= \frac{2 \cdot (3x^2+2x) \cdot (12x^2-18x+4x-6-1)}{(2x-3)^2} = \frac{2 \cdot (3x^2+2x) \cdot (12x^2-14x-7)}{(2x-3)^2}
 \end{aligned}$$

Ecris l'équation de la tangente au graphique de la fonction f au point d'abscisse a .

a) $f(x) = x^2 + 5x - 2$ $a = 1$

$$T \equiv y - f(1) = f'(1) \cdot (x - 1)$$

$$\diamond f(1) = 1^2 + 5 \cdot 1 - 2 = 4$$

$$\diamond f'(x) = 2x + 5$$

$$\diamond f'(1) = 2 \cdot 1 + 5 = 7$$

$$T \equiv y - 4 = 7 \cdot (x - 1)$$

$$T \equiv y = 7x - 3$$

b) $f(x) = x^2(x + 2)$ $a = -1$

$$T \equiv y - f(-1) = f'(-1) \cdot (x - (-1))$$

$$\diamond f(-1) = (-1)^2 \cdot (-1 + 2) = 1$$

$$\diamond f'(x) = 2x \cdot (x + 2) + x^2 \cdot 1 = 2x^2 + 4x + x^2 = 3x^2 + 4x$$

$$\diamond f'(-1) = 3 \cdot (-1)^2 + 4 \cdot (-1) = -1$$

$$T \equiv y - 1 = -1 \cdot (x + 1)$$

$$T \equiv y = -x$$

c) $f(x) = x^3 - 3x^2$ $a = 2$

$$T \equiv y - f(2) = f'(2) \cdot (x - 2)$$

$$\diamond f(2) = 2^3 - 3 \cdot 2^2 = -4$$

$$\diamond f'(x) = 3x^2 - 6x$$

$$\diamond f'(2) = 3 \cdot 2^2 - 6 \cdot 2 = 0$$

$$T \equiv y + 4 = 0$$

$$T \equiv y = -4$$

$$d) f(x) = \frac{4x+1}{x-3}$$

$$a = 0$$

$$T \equiv y - f(0) = f'(0) \cdot (x - 0)$$

$$\diamond f(0) = \frac{-1}{3}$$

$$\diamond f'(x) = \frac{4 \cdot (x-3) - (4x+1) \cdot 1}{(x-3)^2} = \frac{4x-12-4x-1}{(x-3)^2} = \frac{-13}{(x-3)^2}$$

$$\diamond f'(0) = \frac{-13}{9}$$

$$T \equiv y + \frac{1}{3} = \frac{-13}{9} \cdot x$$

$$T \equiv y = \frac{-13}{9} \cdot x - \frac{1}{3}$$

$$e) f(x) = \sqrt{5x+3}$$

$$a = 3$$

$$T \equiv y - f(3) = f'(3) \cdot (x - 3)$$

$$\diamond f(3) = \sqrt{5 \cdot 3 + 3} = \sqrt{18} = 3\sqrt{2}$$

$$\diamond f'(x) = \left((5x+3)^{\frac{1}{2}} \right)' = \frac{1}{2} \cdot (5x+3)^{-\frac{1}{2}} \cdot 5 = \frac{5}{2 \cdot \sqrt{5x+3}}$$

$$\diamond f'(3) = \frac{5}{2 \cdot \sqrt{5 \cdot 3 + 3}} = \frac{5}{6\sqrt{2}} = \frac{5\sqrt{2}}{12}$$

$$T \equiv y - 3\sqrt{2} = \frac{5\sqrt{2}}{12} \cdot (x - 3)$$

$$T \equiv y = \frac{5\sqrt{2}}{12} x + \frac{17\sqrt{2}}{4}$$

$$f) f(x) = 9x^3 - 4x^2 + 5x - 1$$

$$a = -3$$

$$T \equiv y - f(-3) = f'(-3) \cdot (x - (-3))$$

$$\diamond f(-3) = 9 \cdot (-3)^3 - 4 \cdot (-3)^2 + 5 \cdot (-3) - 1 = -295$$

$$\diamond f'(x) = 27x^2 - 8x + 5$$

$$\diamond f'(-3) = 27 \cdot (-3)^2 - 8 \cdot (-3) + 5 = 272$$

$$T \equiv y + 295 = 272 \cdot (x + 3)$$

$$T \equiv y = 272x + 521$$

Détermine le sens de variation des fonctions suivantes :

$$f_1(x) = x^3 + 3x^2 - 4$$

$$f'(x) = 3x^2 + 6x$$

$$\text{Racines : } 3x^2 + 6x = 0 \quad \Leftrightarrow \quad 3x \cdot (x + 2) = 0 \quad \Leftrightarrow \quad x = 0 \quad x = -2$$

x		-2		0	
$f'(x)$	+	0	-	0	+
$f(x)$		MAX		MIN	

$$\diamond x = -2 \quad y = (-2)^3 + 3 \cdot (-2)^2 - 4 = 0$$

$$\Rightarrow \text{MAX en } (-2; 0)$$

$$\diamond x = 0 \quad y = 0^3 + 3 \cdot 0^2 - 4 = -4$$

$$\Rightarrow \text{MIN en } (0; -4)$$

$$f''(x) = 6x + 6$$

$$\text{Racine : } 6x + 6 = 0 \quad \Leftrightarrow \quad x = -1$$

x		-1	
$f''(x)$	-	0	+
$f(x)$	\cap	PI	\cup

$$\diamond x = -1 \quad y = (-1)^3 + 3 \cdot (-1)^2 - 4 = -1 + 3 - 4 = -2$$

$$\Rightarrow \text{PI en } (-1; -2)$$

$$f_2(x) = -3x^2 + 4x + 5$$

$$f'(x) = -6x + 4$$

Racine : $-6x + 4 = 0 \Leftrightarrow x = \frac{2}{3}$

x		$\frac{2}{3}$	
$f'(x)$	+	0	-
$f(x)$		MAX	

$$\diamond x = \frac{2}{3} \quad y = -3 \cdot \left(\frac{2}{3}\right)^2 + 4 \cdot \frac{2}{3} + 5 = \frac{-4}{3} + \frac{8}{3} + 5 = \frac{19}{3}$$

$$\Rightarrow \text{MAX en } \left(\frac{2}{3}; \frac{19}{3}\right)$$

$$f''(x) = -6$$

Pas de racine

x	
-6	- - -

Pas de PI

$$f_3(x) = -x^3 + 3x^2 + 1$$

$$f'(x) = -3x^2 + 6x$$

$$\text{Racines : } -3x^2 + 6x = 0 \quad \Leftrightarrow \quad 3x \cdot (-x + 2) = 0 \quad \Leftrightarrow \quad x = 0 \quad x = 2$$

x		0		2	
$f'(x)$	-	0	+	0	-
$f(x)$		MIN		MAX	

$$\diamond x = 0 \quad y = -0^3 + 3 \cdot 0^2 + 1 = 1$$

\Rightarrow MIN en (0; 1)

$$\diamond x = 2 \quad y = -2^3 + 3 \cdot 2^2 + 1 = -8 + 12 + 1 = 5$$

\Rightarrow MAX en (2; 5)

$$f''(x) = -6x + 6$$

$$\text{Racine : } -6x + 6 = 0 \quad \Leftrightarrow \quad x = 1$$

x		1	
$f''(x)$	+	0	-
$f(x)$	∪	PI	∩

$$\diamond x = 1 \quad y = -1^3 + 3 \cdot 1^2 + 1 = -1 + 3 + 1 = 3$$

\Rightarrow PI en (1; 3)

$$f_4(x) = (x + 1) \cdot (x - 1)^3$$

$$\begin{aligned} f'(x) &= (x + 1)' \cdot (x - 1)^3 + (x + 1) \cdot ((x - 1)^3)' = 1 \cdot (x - 1)^3 + (x + 1) \cdot 3 \cdot (x - 1)^2 \cdot 1 \\ &= (x - 1)^2 \cdot ((x - 1) + 3 \cdot (x + 1)) = (x - 1)^2 \cdot (x - 1 + 3x + 3) \\ &= (x - 1)^2 \cdot (4x + 2) \end{aligned}$$

$$\text{Racines : } (x - 1)^2 \cdot (4x + 2) = 0 \quad \Leftrightarrow x = 1 \quad x = \frac{-1}{2}$$

x		$\frac{-1}{2}$		1	
$(x - 1)^2$	+	+	+	0	+
$4x + 2$	-	0	+	+	+
$f'(x)$	-	0	+	0	+
$f(x)$		<i>MIN</i>		/	

$$\diamond x = \frac{-1}{2} \quad y = \left(\frac{-1}{2} + 1\right) \cdot \left(\frac{-1}{2} - 1\right)^3 = \frac{1}{2} \cdot \frac{-27}{8} = \frac{-27}{16}$$

$$\Leftrightarrow \text{MIN en } \left(\frac{-1}{2}; \frac{-27}{16}\right)$$

$$\begin{aligned} f''(x) &= 2(x - 1)(4x + 2) + (x - 1)^2 \cdot 4 = 2(x - 1)2(2x + 1) + (x - 1)^2 \cdot 4 \\ &= 4(x - 1)(2x + 1 + x - 1) = 4(x - 1)3x = 12x(x - 1) \end{aligned}$$

$$\text{Racines : } 12x(x - 1) = 0 \quad \Leftrightarrow x = 0 \quad x = 1$$

x		0		1	
$12x$	-	0	+	+	+
$x - 1$	-	-	-	0	+
$f''(x)$	+	0	-	0	+
$f(x)$	∪	<i>PI</i>	∩	<i>PI</i>	∪

$$\diamond x = 0 \quad y = (0 + 1) \cdot (0 - 1)^3 = 1 \cdot (-1) = -1$$

⇒ PI en (0; -1)

$$\diamond x = 1 \quad y = (1 + 1) \cdot (1 - 1)^3 = 0$$

⇒ PI en (1; 0)

$$f_5(x) = \frac{x^2 - 5x + 10}{x - 2}$$

$$\begin{aligned} f'(x) &= \frac{(x^2 - 5x + 10)' \cdot (x - 2) - (x^2 - 5x + 10) \cdot (x - 2)'}{(x - 2)^2} \\ &= \frac{(2x - 5) \cdot (x - 2) - (x^2 - 5x + 10) \cdot 1}{(x - 2)^2} \\ &= \frac{2x^2 - 4x - 5x + 10 - x^2 + 5x - 10}{(x - 2)^2} = \frac{x^2 - 4x}{(x - 2)^2} \end{aligned}$$

Racines :

- $x^2 - 4x = 0 \Leftrightarrow x \cdot (x - 4) = 0 \Leftrightarrow x = 0 \quad x = 4$
- $(x - 2)^2 = 0 \Leftrightarrow x = 2$

x		0		2		4	
$x^2 - 4x$	+	0	-	-	-	0	+
$(x - 2)^2$	+	+	+	0	+	+	+
$f'(x)$	+	0	-	/	-	0	+
$f(x)$		MAX		/		MIN	

$$\diamond x = 0 \quad y = -5$$

⇒ MAX en (0; -5)

$$\diamond x = 4 \quad y = \frac{4^2 - 5 \cdot 4 + 10}{4 - 2} = \frac{6}{2} = 3$$

⇒ MIN en (4; 3)

$$f_6(x) = 4x^4 - 6x^2$$

$$f'(x) = 16x^3 - 12x$$

$$\text{Racines : } 16x^3 - 12x = 0 \quad \Leftrightarrow 4x \cdot (4x^2 - 3) = 0 \quad \Leftrightarrow x = 0 \quad x = \frac{-\sqrt{3}}{2} \quad x = \frac{\sqrt{3}}{2}$$

x		$\frac{-\sqrt{3}}{2}$		0		$\frac{\sqrt{3}}{2}$	
$4x$	-	-	-	0	+	+	+
$4x^2 - 3$	+	0	-	-	-	0	+
$f'(x)$	-	0	+	0	-	0	+
$f(x)$		MIN		MAX		MIN	

$$\diamond x = \frac{-\sqrt{3}}{2} \quad y = 4 \left(\frac{-\sqrt{3}}{2} \right)^4 - 6 \left(\frac{-\sqrt{3}}{2} \right)^2 = \frac{9}{4} - \frac{9}{2} = \frac{-9}{4}$$

⇒ MIN en $\left(\frac{-\sqrt{3}}{2}; \frac{-9}{4} \right)$

$$\diamond x = 0 \quad y = 0$$

⇒ MAX en (0; 0)

$$\diamond x = \frac{\sqrt{3}}{2} \quad y = \frac{-9}{4}$$

⇒ MIN en $\left(\frac{\sqrt{3}}{2}; \frac{-9}{4} \right)$

$$\begin{aligned} f''(x) &= \frac{(2x - 4)(x - 2)^2 - (x^2 - 4x) \cdot 2(x - 2)}{((x - 2)^2)^2} \\ &= \frac{2(x - 2)(x - 2)^2 - (x^2 - 4x)2(x - 2)}{(x - 2)^4} \\ &= \frac{2(x - 2)((x - 2)^2 - (x^2 - 4x))}{(x - 2)^4} = \frac{2(x^2 - 4x + 4 - x^2 + 4x)}{(x - 2)^3} = \frac{2 \cdot 4}{(x - 2)^3} \\ &= \frac{8}{(x - 2)^3} \end{aligned}$$

Racine : 8 /

$$(x - 2)^3 = 0 \quad \Leftrightarrow x = 2$$

x		2	
8	+	+	+
$(x - 2)^3$	-	0	+
$f''(x)$	-	\neq	+
$f(x)$	\cap		\cup

Pas de PI