$$\tan 2b = \frac{2\tan b}{1-\tan^2 b} = \frac{2.\frac{-\sqrt{2}}{4}}{1-\frac{2}{16}} = \frac{-\frac{\sqrt{2}}{2}}{\frac{14}{16}} = \frac{-4\sqrt{2}}{7}.$$

f) $\cot(a+b) = \frac{1}{\tan(a+b)} = \frac{1-\tan a.\tan b}{\tan a+\tan b}$

$\tan a = \frac{\sin a}{\cos a} = \frac{3/5}{4/5} = \frac{3}{4}$ et $\tan b = -\frac{\sqrt{2}}{4}$ (voir (e)).

$$\cot(a+b) = \frac{1-\frac{3}{4}.\frac{-\sqrt{2}}{4}}{\frac{3}{4}-\frac{\sqrt{2}}{4}} = \frac{\frac{16+3\sqrt{2}}{16}}{\frac{3-\sqrt{2}}{4}} = \frac{16+3\sqrt{2}}{4.(3-\sqrt{2})}.$$

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4. a) $\cos(a-b).\cos(a+b)$

$= (\cos a.\cos b + \sin a.\sin b).(\cos a.\cos b - \sin a.\sin b)$

$= \cos^2 a.\cos^2 b - \sin^2 a.\sin^2 b$

$= \cos^2 a.(1-\sin^2 b) - (1-\cos^2 a).\sin^2 b$

$= \cos^2 a - \cancel{\cos^2 a.\sin^2 b} - \sin^2 b + \cancel{\cos^2 a.\sin^2 b} = \cos^2 a - \sin^2 b.$

b) $\tan\left(\frac{\pi}{4}+x\right).\tan\left(\frac{\pi}{4}-x\right) = \frac{1+\tan x}{1-\tan x}.\frac{1-\tan x}{1+\tan x} = 1.$

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5. $\tan(2a-b) = \frac{\tan 2a - \tan b}{1+\tan 2a.\tan b}$ $\qquad \tan 2a = \frac{2.2}{1-2^2} = -\frac{4}{3}$

$$= \frac{-\frac{4}{3}-(-1)}{1+\frac{-4}{3}.(-1)} = \frac{-\frac{1}{3}}{\frac{7}{3}} = -\frac{1}{7}.$$

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6. Comme $\frac{\pi}{6} = 2.\frac{\pi}{12}$, on a : $\tan\frac{\pi}{6} = \frac{2.\tan\frac{\pi}{12}}{1-\tan^2\frac{\pi}{12}}$.

Comme $\tan\frac{\pi}{6} = \frac{\sqrt{3}}{3}$, il faut vérifier que $\frac{\sqrt{3}}{3} = \frac{2.(2-\sqrt{3})}{1-(2-\sqrt{3})^2}$.

Développons le second membre :

$$\frac{4-2\sqrt{3}}{1-(4-4\sqrt{3}+3)} = \frac{4-2\sqrt{3}}{-6+4\sqrt{3}} = \frac{2-\sqrt{3}}{-3+2\sqrt{3}}.\frac{2\sqrt{3}+3}{2\sqrt{3}+3} = \frac{4\sqrt{3}+\cancel{6}-\cancel{6}-3\sqrt{3}}{12-9} = \frac{\sqrt{3}}{3}.$$

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7. Si $t = \tan\frac{x}{2}$, l'équation s'écrit $\quad$ (c.e. $\frac{x}{2} \neq \frac{\pi}{2}+k\pi$)

$\frac{1-t^2}{1+t^2} + \sqrt{3}.\frac{2t}{1+t^2} = 1 \iff \cancel{1}-t^2+2\sqrt{3}t = \cancel{1}+t^2$

$\iff -2t^2+2\sqrt{3}t = 0$

$\iff t.(\sqrt{3}-t) = 0 \rightarrow \cdots$